REPORT DOCUMENTATION PAGE	*Form Approved OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
	FINAL REPORT	01 MAY 2006 - 31 OCT 2007

4. TITLE AND SUBTITLE
A RESEARCH PROGRAM ON THE ASYMPTOTIC DESCRIPTION OF ELECTROMAGNETIC PULSE PROPGATION IN SPATIALLY INHOMOGENEOUS, TEMPORALLY DISPERSIVE, ATTENUATIVE MEDIA

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-06-1-0268

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)
PROFESSOR OUGHSTUN

5d. PROJECT NUMBER
2304/IX

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
UNIVERSITY OF VERMONT
85 SO PROSPECT ST
BURLINGTON VT 05405

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AF OFFICE OF SCIENTIFIC RESEARCH
875 NORTH RANDOLPH STREET ROOM 3112
ARLINGTON VA 22203
DR ARJE NACHMAN /NE

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
DISTRIBUTION STATEMEN A: UNLIMITED

AFRL-SR-AR-TR-07-0444

13. SUPPLEMENTARY NOTES

14. ABSTRACT
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15. SUBJECT TERMS
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16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER *(Include area code)*

FINAL TECHNICAL REPORT
USAF Grant # FA9550-06-1-0268

A Research Program on
The Asymptotic Description of
Electromagnetic Pulse Propagation
In Spatially Inhomogeneous,
Temporally Dispersive, Attenuative Media

(May 1, 2006 – October 31, 2006)

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September 2007

This brief DEPSCoR/AFOSR sponsored research grant has been used to continue the extension of our research effort on the asymptotic theory of ultrawideband electromagnetic pulse propagation through temporally dispersive, attenuative media to spatially inhomogeneous, conducting, gyrotropic media, with particular emphasis on pulsed wave propagation through the ionosphere and magnetosphere. This funded research has provided a detailed asymptotic description of ultrawideband electromagnetic pulse propagation through a semiconducting medium as described by the Debye model with static conductivity. This analysis shows that the presence of conductivity does not eliminate the appearance of the Brillouin precursor that is associated with ultrawideband pulse propagation through a Debye model dielectric and dominates the propagated field evolution as the propagation distance z exceeds a single absorption depth z_d at the input pulse carrier frequency, decaying only as $z^{-1/2}$ as $z \to \infty$. With conductivity present, the Brillouin precursor now experiences some exponential attenuation in addition to its characteristic $z^{-1/2}$ algebraic decay. Nevertheless, our analysis has shown that this exponential attenuation is less than that experienced by the remainder of the pulse. As a consequence, the Brillouin precursor remains as the pulse component that experiences minimal attenuation in a dispersive attenuative medium, even in the presence of static conductivity.

The results of this funded research have been published in the Proceedings of the Fourth IASTED International Conference on Antennas, Radar, and Propagation that was held in Montreal, Canada in May 2007. A copy of this paper is attached to this report.

ULTRAWIDEBAND PULSE PENETRATION IN A DEBYE MEDIUM WITH STATIC CONDUCTIVITY

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ABSTRACT
The high frequency, low power characteristics of ultrawideband/short-pulse signals exhibit uniquely promising features with applications to remote sensing of terrestrial objects from satellites, foliage-penetrating radar, as well as the study of biological tissue exposed to ultrawideband pulses. Indeed, previous studies of ultrawideband electromagnetic pulse propagation through dispersive, nonconducting media has shown the existence of a so-called Brillouin precursor whose peak amplitude only decays algebraically with propagation distance. However, materials such as the ionosphere, foliage and biological tissue exhibit conductivity. In this paper, we show that a Debye-model material with static conductivity does indeed support a Brillouin precursor, but that this precursor now attenuates exponentially with propagation distance and not just algebraically. Nevertheless, we show that it is still advantageous to track the Brillouin precursor in remote sensing applications because its attenuation is less than the attenuation of the main signal.

KEY WORDS
wave penetration, ultrawideband pulse propagation, precursors

1 Introduction

An ultrawideband (UWB) electromagnetic pulse is a pulse whose bandwidth spans tens of megahertz to several gigahertz. In the time domain, this corresponds to signals that have a pulse duration on the order of 100 picoseconds (ps) or less (short-pulse), or signals that have a sufficiently rapid turn-on time also of order 100 ps or less. Ultrawideband technologies are being touted by some as one of the most promising new technologies of our time because their high frequency, low power characteristics offer security, increased penetration capabilities, immunity from multipath interference, precise positioning capability and high data rates. Advances in generating such pulse types will enable ultrawideband pulses to be used in an increasing number of leading-edge technologies such as remote sensing of terrestrial objects from satellites, foliage-penetrating radar, next-generation cellular telephones, as well as the study of biological tissue exposed to ultrawideband pulses. All applications of UWB will involve the propagation of UWB pulses through dispersive material such as the earth's surface, trees, walls or the ionosphere. Previous studies [1] of UWB/short-pulse electromagnetic signals through dispersive, nonconducting material has shown the existence of a Brillouin precursor whose peak amplitude decays only algebraically with propagation distance; a considerable attribute for application purposes. However, many physical materials, such as the ionosphere, foliage and biological tissue, are conductive. The dielectric permittivity of a conductive material possesses a simple pole at the origin $\omega = 0$, which is the very point that provides the peak amplitude of the Brillouin precursor and so it has been suggested that conductivity prevents the formation of a Brillioun precursor. In this paper, we present an analysis of an ultrawideband electromagnetic pulse propagating through a Debye material [2] with static conductivity and we study the effect of conductivity on the formation of the Brillouin precursor.

2 Formulation

Let all of space be occupied by a Debye-type material with static conductivity whose relative complex dielectric permittivity is given by

$$\epsilon_c(\omega) = \epsilon_\infty + \frac{\Delta\epsilon}{1 - i\omega\tau} + i\frac{\sigma}{\omega}, \tag{1}$$

where ω represents angular frequency, ϵ_∞ is the high frequency limit of the dielectric permittivity, $\Delta\epsilon = \epsilon_s - \epsilon_\infty$ where $\epsilon_s \equiv \epsilon(0)$ is the static permittivity of the material, τ is the characteristic relaxation time of the material, and $\sigma = \sigma_0/\epsilon_0$ where σ_0 is the static conductivity of the material and ϵ_0 is the permittivity of free space. Here, we employ the values $\epsilon_\infty = 15.24, \epsilon_s = 23.10$ and $\tau = 1.16 \times 10^{-10}$s which, with $\sigma_0 \approx 1 \times 10^{-10}$mho/m are appropriate parameter values for leafy vegetation [3].

Consider a linearly-polarized, electromagnetic plane-wave traveling in the positive z-direction. Let the electric field on the plane $z = 0$ be a step-function modulated sinusoid of fixed carrier frequency

$$\mathbf{E}(0, t) = u(t)\sin(\omega_c t)\hat{\mathbf{x}}, \tag{2}$$

where $u(t)$ is the Heaviside step-function and ω_c is the fixed carrier frequency of the pulse. For the examples given here, $\omega_c = 2\pi \times 10^6$ rad/s. Then the electric field component of the propagated signal on any plane $z > 0$ is given exactly by the Fourier-Laplace integral representation [1]

$$E(z,t) = -\frac{1}{2\pi}\Re\left\{\int_{ia-\infty}^{ia+\infty} \frac{1}{\omega-\omega_c}\exp\left[\frac{z}{c}\phi(\omega,\theta)\right]d\omega\right\}. \tag{3}$$

where $a > 0$ is greater that the abscissa of absolute convergence for the initial electric field component $E(0,t)$, c is the speed of light in vacuum, and $\phi(\omega,\theta)$ is the complex phase function defined as

$$\phi(\omega,\theta) = i\omega\left[n(\omega)-\theta\right], \tag{4}$$

where $n(\omega) = \sqrt{\mu\epsilon_c(\omega)}$ is the complex index of refraction and $\theta \equiv ct/z$ is a dimensionless space-time parameter. Here, the magnetic permeability is always taken as $\mu = 1$. Note that in the integral representation (3), $\omega = \omega' + i\omega''$ is complex.

We concern ourselves only with the determination of the electric field component because the magnetic field component of the transverse plane-wave can be determined by the relation $\mathbf{B} = \tilde{\mathbf{k}}(\omega) \times \mathbf{E}$, where $\tilde{\mathbf{k}}(\omega) = (w/c)n(\omega)\hat{\mathbf{z}}$ is the complex wavevector. Note that our initial pulse is ultrawideband in the sense that its spectrum decays as ω^{-1} as $\omega \to \infty$. Unfortunately, the integral (3) has no known closed-form solution valid for all values of θ (except in vacuum). Instead, we apply uniform asymptotic techniques to provide an analytic approximation to the propagated field, the accuracy of this approximation increasing with increasing propagation distance.

3 Topology of the Complex Phase Function

The complex phase function of a Debye medium with static conductivity

$$\phi(\omega,\theta) = i\omega\left[\sqrt{\epsilon_\infty + \frac{\Delta\epsilon}{1-i\omega\tau} + i\frac{\sigma}{\omega}} - \theta\right] \tag{5}$$

is a multi-valued function with four branch points, one located at the origin and the remaining located at $\omega_{1,2,3}$ where

$$\omega_1 = -i\frac{\epsilon_s+\sigma\tau}{2\epsilon_\infty\tau}\left[1+\sqrt{1-\frac{4\epsilon_\infty\tau\sigma}{\epsilon_s+\sigma\tau^2}}\right] \approx -i\frac{\sigma}{\epsilon_s+\sigma\tau}, \tag{6a}$$

$$\omega_2 = -i/\tau, \tag{6b}$$

$$\omega_3 = -i\frac{\epsilon_s+\sigma\tau}{2\epsilon_\infty\tau}\left[1-\sqrt{1-\frac{4\epsilon_\infty\tau\sigma}{\epsilon_s+\sigma\tau^2}}\right] \approx -i\frac{\epsilon_s+\sigma\tau}{\epsilon_\infty\tau}. \tag{6c}$$

Note that the radicand appearing in (6) is always positive for physically reasonable parameter values $\epsilon_s, \epsilon_\infty, \sigma$ and τ. The complex phase function is made single-valued by construction of two branch cuts along the imaginary axis: one connects the origin and ω_2, the other connects ω_1 and ω_3. On this branch, $\Re\{\phi(\omega,\theta)\}$ is negative, as characteristic of a lossy medium.

For values of $\theta \leq \sqrt{\epsilon_\infty}$, the contour of integration appearing in the integral representation of the propagated field (3) may be completed in the upper half of the complex ω-plane. For sufficiently large $|\omega|$, the real part of the complex phase function behaves as

$$X(\omega,\theta) \equiv \Re\{\phi(\omega,\theta)\} = -\omega''\left(\sqrt{\epsilon_\infty}-\theta\right) + \mathcal{O}(1), \tag{7}$$

and because $\phi(\omega,\theta)$ is analytic in the upper half plane (excluding the real axis), it follows from residue theory that the propagated field is identically zero for $\theta < \sqrt{\epsilon_\infty}$. One may also prove that the propagated field is zero at the space-time point $\theta = \sqrt{\epsilon_\infty}$ by rewritting Eq. (3) as

$$E(z,t) = \frac{\omega_c}{2\pi}\int_{ia-\infty}^{ia+\infty} \frac{1}{\omega^2-\omega_c^2}\exp\left[\frac{z}{c}\phi(\omega,\theta)\right]d\omega \tag{8}$$

and then using a similar argument as above. Hence, in the following discussion we consider the evaluation of Eq. (3) for values of $\theta > \sqrt{\epsilon_\infty}$.

The saddle points of the complex phase function are solutions to the equation

$$\phi'(\omega,\theta) = i\left[n(\omega) - \theta + \omega n'(\omega)\right] = 0, \tag{9}$$

where the prime denotes differentiation with respect to ω. There are four solutions to (9), but one solution lies along the imaginary axis below the branch point ω_3 and may thus be ignored since it is not needed in the remaining analysis. The locations of the other three saddle points may be approximated in the region $|\omega| < 1/\tau$ for $\epsilon_\infty < \theta < \infty$ by the expressions

$$\omega_{SP_1}(\theta) = \begin{cases} i\left(\frac{\epsilon_s(\epsilon_s-\theta^2)}{\tau D} + \frac{\sigma}{\epsilon_s}\right), & \theta < \theta_0; \\ i\left(\frac{\sigma^2}{4\tau D}\right)^{1/3}, & \theta = \theta_0; \\ i\frac{\sigma}{2\epsilon_s}\left(\frac{\theta}{\sqrt{\theta^2-\epsilon_s}} - 1\right), & \theta > \theta_0; \end{cases} \tag{10}$$

$$\omega_{SP_2}(\theta) = \begin{cases} \frac{\sigma}{2\epsilon_s}\left(-i - \frac{\theta}{\sqrt{\epsilon_s-\theta^2}}\right), & \theta < \theta_0; \\ e^{i7\pi/6}\left(\frac{\sigma^2}{4\tau D}\right)^{1/3}, & \theta = \theta_0; \\ -i\frac{\sigma}{2\epsilon_s}\left(\frac{\theta}{\sqrt{\theta^2-\epsilon_s}} + 1\right), & \theta > \theta_0; \end{cases} \tag{11}$$

$$\omega_{SP_3}(\theta) = \begin{cases} \frac{\sigma}{2\epsilon_s}\left(-i + \frac{\theta}{\sqrt{\epsilon_s-\theta^2}}\right), & \theta < \theta_0; \\ e^{-i\pi/6}\left(\frac{\sigma^2}{4\tau D}\right)^{1/3}, & \theta = \theta_0; \\ i\left(\frac{\epsilon_s(\epsilon_s-\theta^2)}{\tau D} + \frac{\sigma}{\epsilon_s}\right), & \theta > \theta_0; \end{cases} \tag{12}$$

where

$$D \equiv \epsilon_s(\epsilon_s - 3\epsilon_\infty) + \theta^2(\epsilon_s + \epsilon_\infty), \tag{13}$$

and $\theta_0 = \sqrt{\epsilon_s}$. These expressions show that $\omega_{SP_1}(\theta)$ lies along the positive imaginary axis for all $\theta > \sqrt{\epsilon_\infty}$ and approaches the branch point at the origin as $\theta \to \infty$, while $\omega_{SP_{1,2}}(\theta)$ evolve in the complex lower half plane for $\epsilon_\infty < \theta \leq \theta_0$, coalesce into a second-order saddle point along the

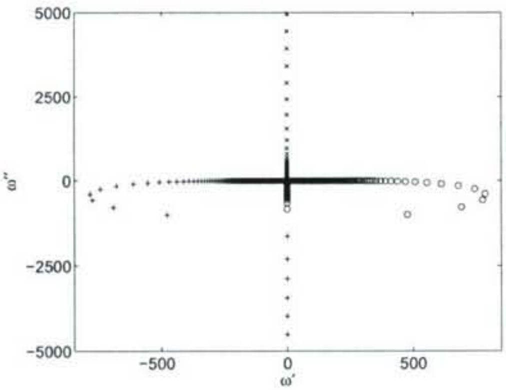

Figure 1. Evolution of the saddle points of $\phi(\omega,\theta)$ for a Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m. The saddle points $\omega_{SP_{1,2,3}}(\theta)$ are depicted by the plus sign, cross and circle, respectively.

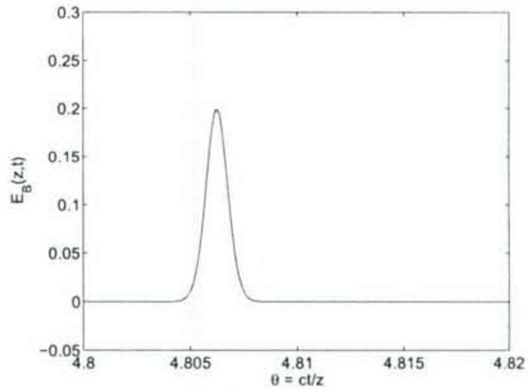

Figure 2. The Brillouin precursor of a step-function modulated sine wave of carrier frequency $\omega_c = 2\pi \times 10^6$ rad/s at a distance of $3z_d$ into a Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m .

negative imaginary axis between the branch points ω_1 and ω_2 at some space-time point $\theta_s \approx \theta_0^+$, and then split again into two first-order saddle points with $\omega_{SP_1}(\theta)$ traveling down the imaginary axis toward the branch point ω_2 as $\theta \to \infty$ while $\omega_{SP_2}(\theta)$ travels upward toward the branch point ω_1 as $\theta \to \infty$. The evolution of the saddle points of the complex phase function for a Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m is shown in Figure 1. The saddle points $\omega_{SP_{1,2,3}}(\theta)$ are depicted by the plus sign, cross and circle, respectively.

4 Asymptotic Approximation

As required by the saddle-point method, the contour appearing in Eq. (3) is deformed into a new path $\mathcal{P}(\theta)$ such that $\mathcal{P}(\theta)$ is continuous in θ and $\mathcal{P}(\theta)$ passes through the dominant (and accessible) saddle points of $\phi(\omega,\theta)$ while remaining in their valleys. For the analysis here, it is sufficient to consider a path $\mathcal{P}(\theta)$ that passes through $\omega_{SP_1}(\theta)$ when $\theta < \theta_0$ and $\omega_{SP_2}(\theta)$ when $\theta > \theta_0$. Of course, in the deformation of the original Bromwich contour into the path $\mathcal{P}(\theta)$, the branch cut extending form $\omega = 0$ to ω_1 is encircled for $\theta > \theta_0$. For the purpose of this paper, the integration around this branch cut is negligible [4].

4.1 Brillouin Precursor

The asymptotic contribution of the saddle points $\omega_{SP_1}(\theta)$ for $\theta < \theta_0$ and $\omega_{SP_2}(\theta)$ for $\theta > \theta_0$ yields the Brillouin precursor, $E_B(z,t)$. Use of the saddle-point method to the integral representation Eq. (3) yields

$$E_B(z,t) = -\frac{1}{2\pi}\Re\left\{\frac{1}{\omega_{SP}-\omega_c}\sqrt{\frac{2c\pi}{-z\phi''(\omega_{SP}(\theta),\theta)}}\right.$$
$$\left.\cdot\exp\left[\frac{z}{c}\phi(\omega_{SP}(\theta),\theta)\right]\right\}, \qquad (14)$$

where

$$\omega_{SP}(\theta) = \begin{cases}\omega_{SP_1}(\theta), & \text{for } \theta < \theta_0;\\ \omega_{SP_3}(\theta), & \text{for } \theta > \theta_0.\end{cases} \qquad (15)$$

An example of the Brillouin precursor in this Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m at a propagation distance of three absorption depths is given in Figure 2. Here, z_d is one absorption depth at the input signal carrier frequency ω_c and is given by $z_d = [(\omega/c)\Im\{n(\omega_c)\}]^{-1}$.

4.2 Signal Contribution

The signal contribution $E_c(z,t)$ to the propagated field solution is due to the simple pole located at ω_c appearing in (3). When $\Re\{\phi(\omega_c,\theta)\} = \Re\{\phi(\omega_{SP}(\theta),\theta)\}$, the effect of the pole on the saddle point $\omega_{SP}(\theta)$ must be taken into account. Also, when the path of integration $\mathcal{P}(\theta)$ crosses the pole, the appropriate residue contribution must be added to the field. A direct application of the uniform theory of Felsen [5] and Felsen and Marcuvitz [6] yields the signal contribution

$$E_c(z,t) =$$
$$-\frac{1}{2\pi}\Re\left\{\pm i\pi\exp\left[\frac{z}{c}\phi(\omega_c,\theta)\right]\operatorname{erfc}\left(\mp i\Delta(\theta)\sqrt{\frac{z}{c}}\right)\right.$$
$$\left.+\frac{1}{\Delta(\theta)}\sqrt{\frac{\pi c}{z}}\exp\left[\frac{z}{c}\phi(\omega_{SP}(\theta),\theta)\right]\right\}, \Im\{\Delta(\theta)\} \gtrless 0; \qquad (16)$$

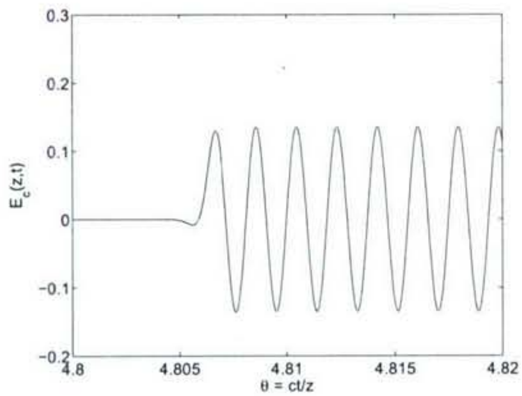

Figure 3. The signal contribution of a step-function modulated sine wave of carrier frequency $\omega_c = 2\pi \times 10^6$ rad/s at a distance of $3z_d$ into a Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m .

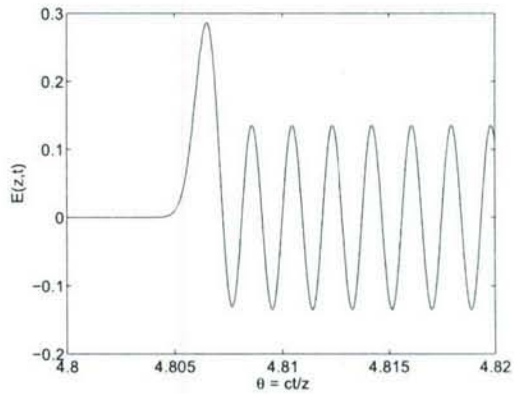

Figure 4. Asymptotic (dashed) and numerical (solid) solutions to the propagated electric field component of a step-function modulated sine wave of carrier frequency $\omega_c = 2\pi \times 10^6$ rad/s at a distance of $3z_d$ into a Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m.

$$\begin{aligned}
E_c(z,t) = & \\
& -\frac{1}{2\pi}\Re\left\{\pm i\pi \exp\left[\frac{z}{c}\phi(\omega_c,\theta)\right] \operatorname{erfc}\left(\mp i\Delta(\theta)\sqrt{\frac{z}{c}}\right)\right. \\
& \left.+\frac{1}{\Delta(\theta)}\sqrt{\frac{\pi c}{z}}\cdot \exp\left[\frac{z}{c}\phi(\omega_{SP}(\theta),\theta)\right]\right\} \\
& -\Re\left\{i\exp\left[\frac{z}{c}\phi(\omega_c,\theta)\right]\right\}, \Im\left\{\Delta(\theta)\right\} = 0;
\end{aligned} \tag{17}$$

where

$$\Delta(\theta) = \sqrt{\phi(\omega_{SP}(\theta),\theta) - \phi(\omega_c,\theta)}, \tag{18}$$

with erfc (ξ) denoting the complementary error function, and $\omega_{SP}(\theta)$ is given in (15). An example of the signal contribution in this Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m at a propagation distance of three absorption depths is given in Figure 3.

4.3 The Total Field

The asymptotic approximation of the propagated electric field component on any plane $z > 0$ is given by the sum

$$E(z,t) = E_B(z,t) + E_c(z,t). \tag{19}$$

As an example to illustrate the accuracy of the asymptotic approximation, Figure 4 shows both the asymptotic solution and a numerical solution to the propagated electric field component (3) of a step-function modulated sine wave of applied frequency $\omega_c = 2\pi \times 10^6$ rad/s traveling through a Debye medium with static conductivity $\sigma_0 = 1 \times 10^{-10}$ mho/m at a propagation distance of $3z_d$. The two results are almost indistinguishable.

5 Peak Amplitudes

The penetration capabilites of the Brillouin precusor (14) are now evaluated. We consider three different values for the static conductivity: $\sigma_0 = 1\times10^{-11}, 1\times10^{-10}, 1\times10^{-9}$ mho/m. For each value of conductivity, the peak amplitude of the Brillouin precursor is recorded at the relative distances $z/z_d = 1$ to 10 in increments of one and from $z/z_d = 10$ to 100 in increments of ten, into the Debye material. A cubic spline is fit to the three sets of data points and are shown in Figure 5 by the dashed, dotted and dashed-dotted curves, respectively. The lower solid curve denotes the exponential attenuation $\exp[-z/z_d]$ experienced by the carrier frequency of the pulse. The power of the observed peak amplitude decay is found by taking the logarithm of the peak amplitude values and the logarithm of the relative distances. The average slope of these curves is calculated and then plotted in Figure 6. The average slopes for the three levels of static conductivity $\sigma_0 = 1 \times 10^{-11}, 1 \times 10^{-10}, 1 \times 10^{-9}$ mho/m are represented by the dashed, dotted and dashed-dotted curves, respectively. Evident in these figures is the rapid decay of the carrier frequency of the pulse and the minimal impact that the static conductivity (at least for these values) has on the peak amplitude values of the Brillouin precursor. Hence, it would be advantageous to track the Brillouin precursor rather than the carrier frequency in remote sensing applications through this Debye medium with these levels of static conductivity.

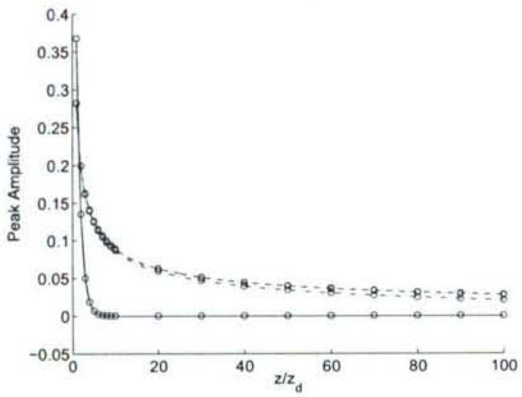

Figure 5. Peak amplitude values of the Brillouin precusor plotted as a function of distance for three different values for the static conductivity: $\sigma_0 = 1 \times 10^{-11}, 1 \times 10^{-10}, 1 \times 10^{-9}$ mho/m given by the dashed, dotted and dashed-dotted curves, respectively. The lower black curve denotes the exponential attenuation $\exp\left[-z/z_d\right]$ experienced by the carrier frequency of the pulse.

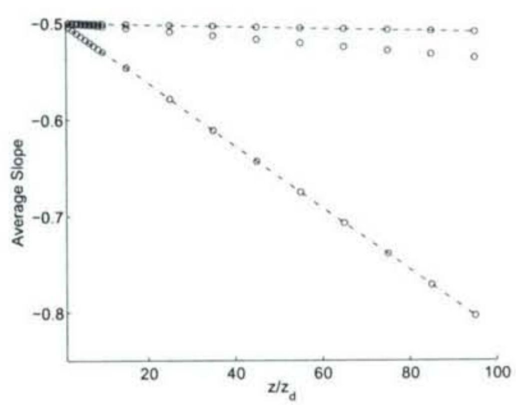

Figure 6. The average slopes of the logarithm of the data presented in Figure 5.

6 Conclusion

An asymptotic approximation to the propagated electric field component of a step-function modulated sine wave through a Debye medium with static conductivity has been presented here. The accuracy of this approximation increases with increasing propagation distance. The asymptotic solution consists of two contributions: the Brillouin precursor and the main signal contribution. The peak amplitude points of these contributions are plotted as functions of relative propagation distance. Both components have a peak amplitude point that decays exponentially, but the peak amplitude point of the Brillouin precursor decays less rapidly. For this reason, it may be advantageous to track the Brillouin precursor rather than the carrier frequency in remote sensing applications through this Debye medium with these levels of static conductivity.

7 Acknowledgements

The research presented in this paper has been supported under AFOSR grant # 000020247.

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[4] For the material parameters used here, there is a small theta domain in which the path of integration must pass through all three saddle points before $\omega_{SP_2}(\theta)$ and $\omega_{SP_3}(\theta)$ coalesce. The fact that the integration around the branch cut is negligible has not been addressed here due to length constraints. However, the accuracy of the asymptotic results with numerical results does much to support this claim.

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